

tion; C, specific heat of material; u, moisture content of material; t, temperature; R, outer radius of cylindrical specimen; x, variable radius of cylinder. Indices: m refers to parameters of surrounding medium; 0 refers to initial state of material.

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#### THERMOELECTRIC AND GALVANOMAGNETIC PROPERTIES OF SYSTEMS WITH MUTUALLY PENETRATING COMPONENTS

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The effective coefficients of thermal conductivity, electrical conductivity, thermal emf, the effective Hall mobility, and the effective Hall coefficient are determined. The analytical dependences obtained are compared with experimental results for a Bi-Cd alloy.

#### Thermoelectrical Properties

The equations for the current density  $\vec{j}_e$  and the heat flux density (energy)  $\vec{j}_q$  in a homogeneous substance under the superposition of electrical and thermal conductivities have the form [1]

$$\vec{j}_e = \sigma \vec{E} - \alpha \sigma \vec{\nabla} T, \quad (1)$$

$$\vec{j}_q = \alpha T \vec{j}_e - \lambda \vec{\nabla} T. \quad (2)$$

The thermal emf coefficient  $\alpha$  is determined from (1) for  $\vec{j}_e = 0$  and  $\vec{\nabla} T \neq 0$ , i.e.,

$$\sigma \vec{E} - \alpha \sigma \vec{\nabla} T = 0. \quad (3)$$

The coefficient of electrical conductivity  $\sigma$  is determined from (1) for  $\vec{\nabla} T = 0$ , and the coefficient of thermal conductivity  $\lambda$  is determined from (2) for  $\vec{j}_e = 0$ .

Let us determine the coefficients  $\alpha$ ,  $\sigma$ ,  $\lambda$  for a two-component layered system (Fig. 1a) when  $\vec{j}_e$  and  $\vec{j}_q$  are directed parallel to the layers along the X axis. The equivalent circuit for this structure is shown in Fig. 1b.

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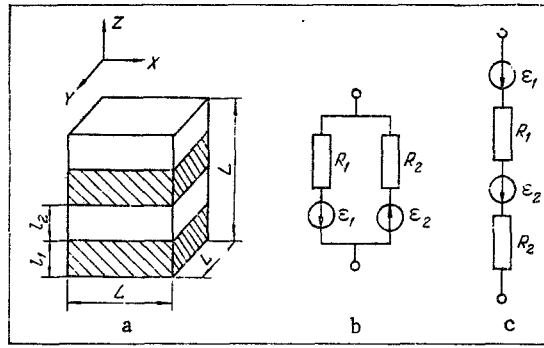


Fig. 1. To calculate the properties of a layered system: a) model of the layered structure; b, c) equivalent circuit of the structure to determine the voltage drop of the electrical field parallel and perpendicular to the layers.

In this case the expression for the current density  $\langle \vec{j}_e \rangle$  passing through the layer can be written in the form

$$\langle \vec{j}_e \rangle = m_1 \langle \vec{j}_{e1} \rangle + m_2 \langle \vec{j}_{e2} \rangle, \quad (4)$$

where  $m_1$  and  $m_2$  are the volume concentrations of the first and second components, respectively. The angular brackets  $\langle \rangle$  denote the average over the volume. The subscripts 1 and 2 will refer, here and henceforth, to the first and second components, respectively, and  $\langle \vec{j}_{ei} \rangle$  is the current density passing through the  $i$ -th component ( $i = 1, 2$ ):

$$\langle \vec{j}_{ei} \rangle = \sigma_i \langle \vec{E} \rangle - \alpha_i \sigma_i \langle \vec{\nabla} T \rangle. \quad (5)$$

Substituting (5) into (4), we obtain

$$\vec{j}_e = (\sigma_1 m_1 + \sigma_2 m_2) \langle \vec{E} \rangle - (\alpha_1 \sigma_1 m_1 + \alpha_2 \sigma_2 m_2) \langle \vec{\nabla} T \rangle. \quad (6)$$

Let us introduce the parameters  $\sigma_{\parallel}, \alpha_{\parallel}$ , the effective coefficients of electrical conductivity and thermal emf of the layered system when the layers are parallel to the fluxes, and let us write for  $\langle \vec{j}_e \rangle$

$$\langle \vec{j}_e \rangle = \sigma_{\parallel} \langle \vec{E} \rangle - \alpha_{\parallel} \sigma_{\parallel} \langle \vec{\nabla} T \rangle. \quad (7)$$

Taking account of (3), we determined  $\alpha_{\parallel}, \sigma_{\parallel}$  from (6) and (7)

$$\alpha_{\parallel} = (\alpha_1 \sigma_1 m_1 + \alpha_2 \sigma_2 m_2) (\sigma_1 m_1 + \sigma_2 m_2)^{-1}, \quad (8)$$

$$\sigma_{\parallel} = \sigma_1 m_1 + \sigma_2 m_2. \quad (9)$$

It is seen from (4) that when the total current density  $\langle \vec{j}_e \rangle$  equals zero the current in the components is not zero for  $\vec{\nabla} T \neq 0$ , i.e., there exists a circulation current caused by the difference in the thermoelectrical properties of the components.

The electromotive force in the circulation current loop equals (see Fig. 1b)

$$\varepsilon = (\alpha_2 - \alpha_1) \Delta T, \quad (10)$$

where  $\Delta T$  is the temperature difference between the isotherms bounding the layered system and perpendicular to the X axis.

According to the second Kirchhoff law

$$\varepsilon = I_1 R_1 + I_2 R_2. \quad (11)$$

Here  $I_1$  and  $I_2$  are the total circulation currents flowing through the first and second components, respectively;  $R_1 = L/(\sigma_1 S_1)$  and  $R_2 = L/(\sigma_2 S_2)$  are the resistances of the first and second components to the electric current;  $S_1 = n_1 l_1 L$ ,  $S_2 = n_2 l_2 L$ ;  $n_i$  is the number of

layers of the first component of height  $l_1$ ;  $n_2$  is the number of layers of the second component of height  $l_2$  (see Fig. 1a). In the case  $I_1 = I_2$ , then taking account of (10) we determine from (11)

$$I_1 = \frac{(\alpha_2 - \alpha_1) \Delta T}{R_1 + R_2} \quad (12)$$

in the loop.

From (12) the expression for the circulation current density can be written in the form ( $I_1 = \langle \vec{j}_{e1} \rangle S_1$ ):

$$\langle \vec{j}_{e1} \rangle = (\alpha_2 - \alpha_1) \sigma_1 \sigma_2 m_2 (m_1 \sigma_1 + m_2 \sigma_2)^{-1} \langle \nabla T \rangle. \quad (13)$$

The expression for the total heat flux density (energy) passing through the layers becomes, analogously to (5),

$$\langle \vec{j}_q \rangle = (\alpha_1 T \langle \vec{j}_{e1} \rangle - \lambda_1 \langle \nabla T \rangle) m_1 + (\alpha_2 T \langle \vec{j}_{e2} \rangle - \lambda_2 \langle \nabla T \rangle) m_2. \quad (14)$$

Hence, if  $\langle \vec{j}_e \rangle = 0$ , then  $\langle \vec{j}_{e1} \rangle m_1 = - \langle \vec{j}_{e2} \rangle m_2$ , and (14) can be rewritten

$$\langle \vec{j}_q \rangle = (\alpha_1 - \alpha_2) T \langle \vec{j}_{e1} \rangle m_1 - (\lambda_1 m_1 + \lambda_2 m_2) \langle \nabla T \rangle. \quad (15)$$

Let us rewrite (15) taking (13) into account

$$\langle \vec{j}_q \rangle = \left[ \frac{(\alpha_2 - \alpha_1)^2 \sigma_1 \sigma_2 m_1 m_2}{m_1 \sigma_1 + m_2 \sigma_2} T + \lambda_1 m_1 + \lambda_2 m_2 \right] \langle \nabla T \rangle. \quad (16)$$

We therefore obtain the following effective coefficient of thermal conductivity of the layered system  $\lambda_{||}$  when the layers are parallel  $\langle \vec{j}_q \rangle$ :

$$\lambda_{||} = \frac{(\alpha_2 - \alpha_1)^2 \sigma_1 \sigma_2 m_1 m_2}{m_1 \sigma_1 + m_2 \sigma_2} T + \lambda_1 m_1 + \lambda_2 m_2. \quad (17)$$

It is seen from (17) that because of the circulation currents an additional thermal conductivity occurs which equals

$$\Delta \lambda_{||} = \frac{(\alpha_2 - \alpha_1)^2 \sigma_1 \sigma_2 m_1 m_2}{m_1 \sigma_1 + m_2 \sigma_2} T. \quad (18)$$

If  $\alpha_1 = \alpha_2$ , then the effective coefficient of thermal conductivity is

$$\lambda_{||} = \lambda_1 m_1 + \lambda_2 m_2. \quad (19)$$

Now let us examine the case when the fluxes  $\vec{j}_e$  and  $\vec{j}_q$  are directed perpendicularly to the layers along the Z axis. The equivalent circuit in this case is shown in Fig. 1c.

The total current density passing perpendicular to the layers equals

$$\langle \vec{j}_e \rangle = \langle \vec{j}_{e1} \rangle = \langle \vec{j}_{e2} \rangle. \quad (20)$$

Hence,

$$\langle \vec{E} \rangle = m_1 \langle \vec{E}_1 \rangle + m_2 \langle \vec{E}_2 \rangle. \quad (21)$$

Analogously, for the heat flux

$$\langle \vec{j}_q \rangle = \langle \vec{j}_{q1} \rangle = \langle \vec{j}_{q2} \rangle, \quad \langle \nabla T \rangle = m_1 \langle \nabla T_1 \rangle + m_2 \langle \nabla T \rangle. \quad (22)$$

It is seen from (20) that no circulation currents occur in the system in this case, since if  $\langle \vec{j}_e \rangle = 0$ , then  $\langle \vec{j}_{e1} \rangle = \langle \vec{j}_{e2} \rangle = 0$ . Thus,  $\langle \vec{E}_1 \rangle$  and  $\langle \vec{E}_2 \rangle$  equal

$$\langle \vec{E}_1 \rangle = \alpha_1 \langle \nabla T_1 \rangle; \quad \langle \vec{E}_2 \rangle = \alpha_2 \langle \nabla T_2 \rangle, \quad (23)$$

then we obtain from (23)

$$\langle \vec{E} \rangle = m_1 \alpha_1 \langle \nabla T_1 \rangle + m_2 \alpha_2 \langle \nabla T_2 \rangle. \quad (24)$$

The effective coefficient of the thermal emf  $\alpha_{\perp}$  of the layered system for a direction  $\langle \vec{j}_e \rangle$  perpendicular to the layers has the form

$$\alpha_{\perp} = m_1 \alpha_1 \frac{\langle \nabla T_1 \rangle}{\langle \nabla T \rangle} + m_2 \alpha_2 \frac{\langle \nabla T_2 \rangle}{\langle \nabla T \rangle}. \quad (25)$$

Taking into account that

$$m_1 \frac{\langle \nabla T_1 \rangle}{\langle \nabla T \rangle} = \frac{\lambda_2 m_1}{m_1 \lambda_2 + m_2 \lambda_1}; \quad m_2 \frac{\langle \nabla T_2 \rangle}{\langle \nabla T \rangle} = \frac{\lambda_1 m_2}{m_1 \lambda_2 + m_2 \lambda_1}, \quad (26)$$

we obtain

$$\alpha_{\perp} = (\alpha_1 \lambda_2 m_1 + \alpha_2 \lambda_1 m_2) (\lambda_1 m_2 + \lambda_2 m_1)^{-1}, \quad (27)$$

from which it is seen that upon disposition of the layers perpendicular to the streams  $\langle \vec{j}_e \rangle$  and  $\langle \vec{j}_q \rangle$ , the effective coefficient of the thermal emf is independent of the electrical conductivity of the components.

The effective coefficients of electrical conductivity  $\sigma_{\perp}$  and thermal conductivity  $\lambda_{\perp}$  are determined from (20)-(22) in the form

$$\sigma_{\perp} = (m_1 \sigma_1^{-1} + m_2 \sigma_2^{-1} + \Delta \rho)^{-1}; \quad \lambda_{\perp} = (m_1 \lambda_1^{-1} + m_2 \lambda_2^{-1})^{-1}, \quad (28)$$

where  $\Delta \rho = m_1 m_2 (\alpha_2 - \alpha_1)^2 T (\lambda_1 m_2 + \lambda_2 m_1)^{-1}$  is the additional electrical resistivity due to the thermoelectric inhomogeneities.

### Thermoelectrical Properties of a System with Mutually Penetrating Components

The model of a structure in the form of a set of two isotropic spatially mutually penetrating lattices of cubic symmetry was used in [2, 3] for the examination of two-component systems. It was assumed that the initial properties of the components do not vary with the change in concentration. An elementary cell of the model, any face of which can be oriented perpendicular to the gradient of the field potential, is shown in Fig. 2a. In this case, the two faces of the cell which are perpendicular to the gradient are isopotential planes, and the remaining four are planes impermeable to the current line. As is recommended in [3], let us divide the elementary cell by a system of two infinitely thin planes 1-1 and 2-2 which are impermeable to the current lines (Fig. 2a). In this case the equivalent circuit is shown in Fig. 2b.

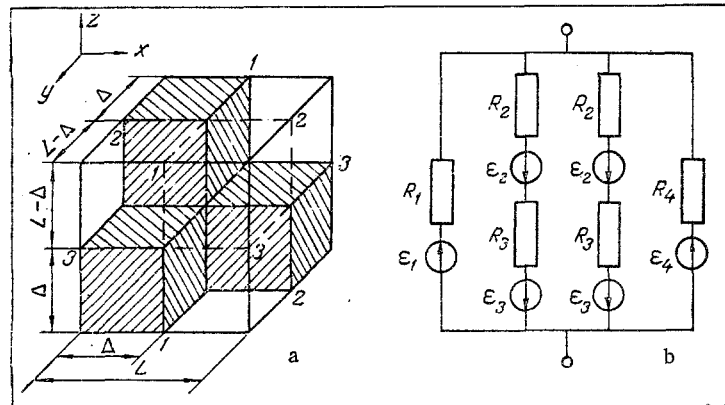


Fig. 2. To calculate the thermoelectrical properties of a system with mutually penetrating components: a) elementary cell of the model; b) equivalent circuit.

In this case the expression for the flux density (heat, electricity)  $\langle \vec{j}_k \rangle$  ( $k = e, q$ ) through the elementary cell can be written in the form

$$\langle \vec{j}_k \rangle = C^2 \langle \vec{j}_{k1} \rangle + (1-C)^2 \langle \vec{j}_{k2} \rangle + 2C(1-C) \langle \vec{j}_{k12} \rangle, \quad (29)$$

where  $\langle \vec{j}_{k1} \rangle$  is the flux density passing through only the first component;  $\langle \vec{j}_{k2} \rangle$  is the flux density passing through only the second component; and  $\langle \vec{j}_{k12} \rangle$  is the flux density passing through the first and second components in sequence.

The value  $C = \Delta/L$  is related uniquely to the  $m_2$  volume concentration of the second component [2, 3]

$$2C^3 - 3C^2 + 1 = m_2. \quad (30)$$

Using (5) and (29), the expression for the effective coefficient of the thermal emf is obtained as

$$\alpha_{\text{eff}} = \rho_{\text{eff}} \left\{ \alpha_1 \sigma_1 C^2 + \alpha_2 \sigma_2 (1-C)^2 + 2 \left[ \frac{\alpha_1 \lambda_2 C + \alpha_2 \lambda_1 (1-C)}{\lambda_1 (1-C) + \lambda_2 C} \right] \left[ \frac{\sigma_1 \sigma_2 C (1-C)}{\sigma_1 (1-C) + \sigma_2 C} \right] \right\}, \quad (31)$$

where  $\rho_{\text{eff}}$  is an effective coefficient of electrical resistivity equal to

$$\rho_{\text{eff}} = \{ \sigma_1 C^2 + \sigma_2 (1-C)^2 + 2C(1-C) [\sigma_1^{-1} + (1-C)\sigma_2^{-1} + \Delta\rho]^{-1} \}^{-1}, \quad (32)$$

where  $\Delta\rho = C(1-C)(\alpha_2 - \alpha_1)^2 T [\lambda_1(1-C) + \lambda_2 C]^{-1}$  is the additional electrical resistivity due to the thermoelectrical inhomogeneities.

The effective coefficient of thermal conductivity for systems with mutually penetrating components  $\lambda_{\text{eff}}$  can be written in the form

$$\lambda_{\text{eff}} = \Delta\lambda_{\text{eff}} + \lambda_{\text{eff}}^0 \quad (33)$$

where  $\Delta\lambda_{\text{eff}}$  is the additional thermal conductivity occurring under the effect of circulation currents caused by the difference between the thermoelectrical properties of the system components;

$$\Delta\lambda_{\text{eff}} = (\alpha_1 - \alpha_2)^2 T \left[ \frac{(1-C)C^2}{\rho_1(1-C+C^2) + \rho_2 C(1-C)} + \frac{C(1-C)^2}{\rho_1 C(1-C) + \rho_2(1-C+C^2)} \right]; \quad (34)$$

$\lambda_{\text{eff}}^0$  is the effective thermal conductivity in the absence of circulation currents [3], which equals

$$\lambda_{\text{eff}}^0 = \lambda_1 C^2 + \lambda_2 (1-C)^2 + 2\lambda_1 \lambda_2 [\lambda_1(1-C) + \lambda_2 C]^{-1}. \quad (35)$$

For a quantitative confirmation of the formulas obtained for  $\alpha_{\text{eff}}$ ,  $\sigma_{\text{eff}}$ , and  $\lambda_{\text{eff}}$ , the computed values were compared with experimental results for a eutectic Bi-Cd alloy [4, 5]. The comparison is presented in Fig. 3 and exhibits satisfactory agreement.

#### Galvanomagnetic Phenomena. Hall Effect

The equation for the current density  $j_e$  in a conductor placed in a magnetic field has the form

$$\langle \vec{j}_e \rangle = \sigma_M E + \sigma_M \mu^H [E \times B], \quad \sigma_M = \sigma [1 + (\mu_d B)^2]^{-1}. \quad (36)$$

If the magnetic field direction  $\vec{B}$  is selected parallel to the Z axis, then (36) can be written in the form

$$\vec{j}_{ex} = \sigma_M \vec{E}_x - \sigma_M \mu^H B_z \vec{E}_y, \quad (37)$$

$$\vec{j}_{ey} = \sigma_M \mu^H B_z \vec{E}_x + \sigma_M \vec{E}_y, \quad (38)$$

$$\vec{j}_{ez} = \sigma E_y. \quad (39)$$

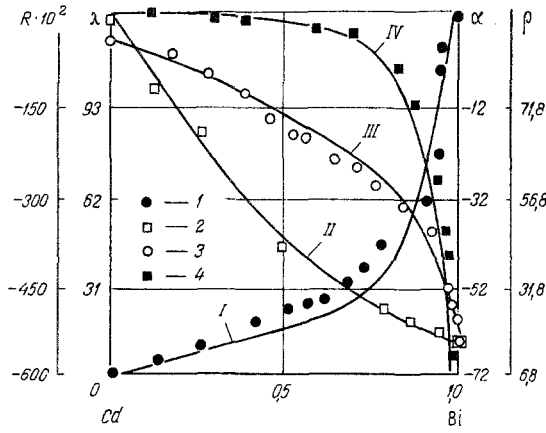


Fig. 3. Concentration dependence of the coefficients: 1) electrical resistivity  $\rho_{\text{eff}}$  [4]; 2) thermal conductivity  $\lambda_{\text{eff}}$  [5]; 3) thermal emf  $\alpha_{\text{eff}}$  [4]; 4) Hall coefficient  $R_{\text{eff}}$  [4] for the eutectic alloy Bi-Cd. Curves I-IV are a computation using (31), (32), (33), (64), respectively; the points are experimental data.  $R$ ,  $\text{cm}^3/\text{C}$ ;  $\lambda$ ,  $\text{W}/\text{m}^\circ\text{K}$ ;  $\alpha$ ,  $\mu\text{V}/^\circ\text{K}$ ;  $\rho$ ,  $\mu\Omega\cdot\text{cm}$ .

The Hall mobility  $\mu^H$  can be determined if it is assumed that  $\vec{j}_{\text{ex}} \neq 0$  in the conductor whose dimensions are bounded, but  $\vec{j}_{\text{ey}} = 0$  and  $\vec{j}_{\text{ez}} = 0$ ; i.e., we obtain from (38)

$$\mu^H = -E_y (\vec{E}_x \vec{B}_z)^{-1}. \quad (40)$$

Substituting (40) into (37), we obtain

$$\vec{j}_{\text{ex}} = \sigma_M [1 + (\mu^H B_z)^2] \vec{E}_x. \quad (41)$$

The Hall coefficient  $R^H$  equals

$$R^H = -\vec{E}_y (\vec{j}_{\text{ex}} \vec{B}_z)^{-1}. \quad (42)$$

Taking (41) into account, (42) becomes

$$R^H = \mu^H \{\sigma_M [1 + (\mu^H B_z)^2]\}^{-1}. \quad (43)$$

In a weak magnetic field ( $\mu^H B_z \ll 1$ ), there follows from (43) that

$$R^H = \mu^H \sigma^{-1}. \quad (44)$$

Let us determine the Hall mobility  $\mu^H$  and Hall coefficient  $R^H$  for a layered system (Fig. 1a). If the layers are perpendicular to the current flowing through the specimen  $\vec{j}_z$  and parallel to the magnetic field  $\vec{B}_x$ , then the boundary conditions are written in the form

$$\langle \vec{j}_z \rangle = \langle \vec{j}_{z1} \rangle = \langle \vec{j}_{z2} \rangle; \quad \langle \vec{j}_y \rangle = m_1 \langle \vec{j}_{y1} \rangle + m_2 \langle \vec{j}_{y2} \rangle = 0, \quad (45)$$

$$\langle \vec{E}_z \rangle = m_1 \langle \vec{E}_{z1} \rangle + m_2 \langle \vec{E}_{z2} \rangle; \quad \langle \vec{E}_y \rangle = \langle \vec{E}_{y1} \rangle = \langle \vec{E}_{y2} \rangle. \quad (46)$$

The mean current density passing through the  $i$ -th component parallel to the X axis ( $i = 1, 2$ )

$$\langle \vec{j}_{zi} \rangle = \sigma_{Mi} \langle \vec{E}_{zi} \rangle - \sigma_{Mi} \mu_i^H \vec{B}_x \langle \vec{E}_{yi} \rangle, \quad (47)$$

and the current parallel to the y axis  $\langle \vec{j}_{yi} \rangle$  equals

$$\langle \vec{j}_{yi} \rangle = \sigma_{Mi} \mu_i^H \vec{B}_x \langle \vec{E}_{zi} \rangle + \sigma_{Mi} \langle \vec{E}_{yi} \rangle. \quad (48)$$

It is seen from (45) that if the galvanomagnetic properties of the components are distinct, then a Hall circulation current parallel to the y axis occurs in the system. The equivalent circuit for the Hall current has the form shown in Fig. 1b.

The effective Hall mobility  $\mu^H$  can be determined from (45) and (46), with (47) and (48) taken into account, in the form

$$\mu^H = \frac{\sigma_{M1} \sigma_{M2} (m_1 \mu_1^H + m_2 \mu_2^H)}{m_1 m_2 (\sigma_{M1} \mu_1^H - \sigma_{M2} \mu_2^H)^2 B_x^2 + (m_1 \sigma_{M1} + m_2 \sigma_{M2}) (m_1 \sigma_{M2} + m_2 \sigma_{M1})}. \quad (49)$$

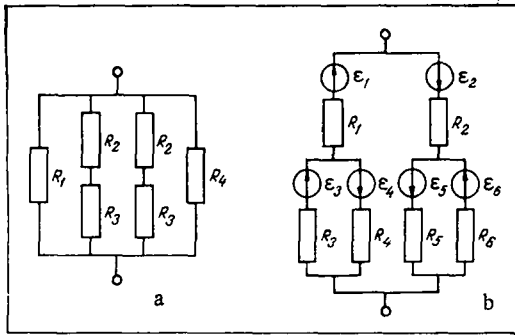


Fig. 4. To calculate the effective Hall mobility of a system with mutually penetrating components: a) equivalent circuit of an elementary cell for a current flowing through the specimen, and b) equivalent circuit of an elementary cell for the Hall voltage.

The first member in the denominator of (49) is much less than the second in a weak magnetic field, and  $\mu^H$  becomes

$$\mu_I^H = \frac{\sigma_1 \sigma_2 (m_1 \mu_1^H + m_2 \mu_2^H)}{(m_1 \sigma_1 + m_2 \sigma_2) (m_1 \sigma_2 + m_2 \sigma_1)}. \quad (50)$$

Taking account of (44), we obtain an expression for the effective Hall coefficient from (50):

$$R_I^H = \frac{m_1 \sigma_1 R_1^H + m_2 \sigma_2 R_2^H}{m_1 \sigma_1 + m_2 \sigma_2}. \quad (51)$$

Let us determine the effective Hall mobility and Hall coefficient when the layers are parallel to the current  $\langle \vec{j}_{ex} \rangle$  and the magnetic field  $\vec{B}_y$ . The equivalent circuit to determine the Hall voltage  $\langle \vec{E}_z \rangle$  is shown in Fig. 1c. In this case the total Hall voltage is

$$\langle \vec{E}_z \rangle = m_1 \langle \vec{E}_{z1} \rangle + m_2 \langle \vec{E}_{z2} \rangle. \quad (52)$$

Hence, according to (40),

$$\langle \vec{E}_{z1} \rangle = -\mu_1^H \vec{B}_y \langle \vec{E}_x \rangle; \quad \langle \vec{E}_{z2} \rangle = -\mu_2^H \vec{B}_y \langle \vec{E}_x \rangle. \quad (53)$$

Taking account of (53), we determine the effective Hall mobility from (52):

$$\mu_{II}^H = m_1 \mu_1^H + m_2 \mu_2^H. \quad (54)$$

We obtain from (54) the expression for the Hall coefficient  $R_{II}^H$  in a weak magnetic field in the form

$$R_{II}^H = \frac{m_1 \sigma_1 R_1^H + m_2 \sigma_2 R_2^H}{m_1 \sigma_1 + m_2 \sigma_2}. \quad (55)$$

Let us determine the effective Hall mobility and Hall coefficient when the layers are parallel to the current passing through the specimen and perpendicular to the magnetic field. The equivalent circuit for the Hall current has the form shown in Fig. 1b. In this case the boundary conditions are written as follows:

$$\langle \vec{j}_x \rangle = m_1 \langle \vec{j}_{x1} \rangle + m_2 \langle \vec{j}_{x2} \rangle; \quad \langle \vec{j}_y \rangle = m_1 \langle \vec{j}_{y1} \rangle + \langle \vec{j}_{y2} \rangle m_2 = 0, \quad (56)$$

$$\langle \vec{E}_x \rangle = \langle \vec{E}_{x1} \rangle = \langle \vec{E}_{x2} \rangle; \quad \langle \vec{E}_y \rangle = \langle \vec{E}_{y1} \rangle = \langle \vec{E}_{y2} \rangle. \quad (57)$$

We obtain the effective Hall mobility from (56) and (57):

$$\mu_{III}^H = \frac{m_1 \sigma_{M1} \mu_1^H + m_2 \sigma_{M2} \mu_2^H}{m_1 \sigma_{M1} + m_2 \sigma_{M2}}. \quad (58)$$

We determine the Hall coefficient  $R_{III}^H$  in a weak magnetic field from (58) in the form

$$R_{III}^H = \frac{m_1 \sigma_1^2 R_1^H + m_2 \sigma_2^2 R_2^H}{m_1 \sigma_1 + m_2 \sigma_2}. \quad (59)$$

## Effective Hall Mobility of Systems with Mutually Penetrating Components

In determining the approximate expression for the effective Hall mobility we divide the elementary cell into separate elements. The equivalent circuit of the elementary cell of the model for the current  $\langle \vec{j}_{ez} \rangle$  flowing through the specimen, obtained by partitioning the elementary cells by the planes 1-1 and 2-2 which are impermeable to the current line  $\langle \vec{j}_{ez} \rangle$ , is shown in Fig. 4a. The equivalent circuit for the Hall circulation current, obtained by partitioning the elementary cell by the plane 3-3, which is impermeable to the Hall circulation current  $\langle \vec{j}_{ey} \rangle$ , and by the isopotential plane 2-2 for the Hall voltage  $\langle \vec{E}_y \rangle$ , is shown in Fig. 4b.

By using the results obtained [(50), (54), (58)], an expression can be written for the effective Hall mobility in systems with mutually penetrating components:

$$\mu_{\text{eff}}^H = \rho_{\text{eff}} [C^2 \mu_1^2 \sigma_I + (1-C)^2 \mu_2^2 \sigma_{II} + C(1-C)(\sigma_I + \sigma_{II}) \mu_{10}^H], \quad (60)$$

where

$$\mu_{10}^H = \frac{\sigma_{M1} \sigma_{M2} [C \mu_1^H + (1-C) \mu_2^H]}{C(1-C)(\sigma_{M1} \mu_1^H - \sigma_{M2} \mu_2^H)^2 B_z^2 + [C \sigma_{M1} + (1-C) \sigma_{M2}] [C \sigma_{M2} + (1-C) \sigma_{M1}]}, \quad (61)$$

$$\sigma_I = C \sigma_{M1} + \sigma_{M1} \sigma_{M2} (1-C) [\sigma_{M1} (1-C) + \sigma_{M2} C]^{-1}, \quad (62)$$

$$\sigma_{II} = (1-C) \sigma_{M2} + \sigma_{M1} \sigma_{M2} C [\sigma_{M1} (1-C) + \sigma_{M2} C]^{-1}. \quad (63)$$

The effective Hall coefficient  $R_{\text{eff}}^H$  in a weak magnetic field can be determined from (61) in the form

$$R_{\text{eff}}^H = \rho_{\text{eff}}^2 \left[ C^2 R_1^H \sigma_1 \sigma_I + (1-C)^2 R_2^H \sigma_2 \sigma_{II} + \frac{C(1-C) \sigma_1 \sigma_2}{C \sigma_2 + (1-C) \sigma_1} (\sigma_I + \sigma_{II}) R_{10}^H \right], \quad (64)$$

where

$$R_{10}^H = \frac{C \sigma_1 R_1^H + (1-C) \sigma_2 R_2^H}{C \sigma_1 + (1-C) \sigma_2}. \quad (65)$$

A comparison of the concentration dependence of  $R_{\text{eff}}^H$  computed by means of (64) with the experimental results for the eutectic alloy Bi-Cd is presented in Fig. 3 and exhibits good qualitative agreement.

### NOTATION

$\vec{j}$ , flux density (heat, electricity);  $\vec{E}$ , electrical field intensity;  $\vec{\nabla}T$ , temperature gradient;  $\vec{B}$ , magnetic field intensity;  $\sigma$ ,  $\alpha$ ,  $\lambda$ , coefficients of electrical conductivity, thermal emf, thermal conductivity;  $\mu_d$ , drift mobility of the carriers;  $\mu^H$ , Hall mobility;  $R^H$ , Hall coefficient;  $m_i$ , volume concentration of the  $i$ -th component;  $\epsilon$ , electromotive force.

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